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Dear Parents/Guardians of students entering AP Calculus BC in September 2024,

This summer, your child will have the opportunity to prevent summer learning loss and to be better prepared for success in AP Calculus BC. He or she will also have the opportunity to earn up to ten extra credit points on the first mathematics test of the 2024-2025 school year.

Note: The assignment is attached to this letter. In order to receive credit, students must show ALL written work and submit it to their teacher by September 18, 2024.

Also, please do not wait until the end of summer to begin these skills.

Dawn Aiello
Director of Mathematics

Name: _____

This packet is due on the first day of school. You will be tested on these concepts on the first week of school.

Limits and Continuity

1. $\lim\limits_{t \to -2} \dfrac{t+2}{t^2+4}$ _____

2. $\lim\limits_{x \to 3} \dfrac{x^3+2x^2-9x-18}{x-3}$ _____

3. $\lim\limits_{x \to 0} \dfrac{\sqrt{4-x}-2}{x}$ _____

4. $\lim\limits_{\theta \to -\infty} \dfrac{\sin\theta}{\theta}$ _____

5. Find the horizontal asymptote (s) of $f(x) = \dfrac{27x-18}{3x+8}$ _____

6. $\displaystyle\lim_{x \to -1} \frac{x^3 + 1}{x - 1}$ _____

7. For what value of a is the function $f(x) = \begin{cases} \dfrac{x^2 - 3x + 2}{x - 2}, & \text{if } x \neq 2 \\ a, & \text{if } x = 2 \end{cases}$ continuous? Show your work to

justify your answer.

8. Find the vertical asymptote(s) of $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 1}$ _____

9. What does it mean for a function to be continuous at x = a? You may describe this in your own words, but also include mathematically sound justification. Identify the different types of discontinuity.

Differentiation

1. What is the slope of the tangent to the curve $3x^2 + y^3 = -37$, when x = 3? _____

2. Find $\dfrac{dy}{dx}$, if $y = 7 + 5^{x^2 + 2x - 1}$ _____

3. $\displaystyle\lim_{h \to 0} \dfrac{\ln(2x + h) - \ln(2x)}{h}$ _____

Use the table below for questions 4 and 5.

x	1	2	3
f(x)	3	0	1
f'(x)	-3	5	-2
g(x)	4	-1	1
g'(x)	-4	3	0

4. If $h(x) = f(x)g(x)$, find h'(2) _____

5. If $k(x) = g(f(x))$, find k(3) _____

6. If $f(x) = 4\sec^2(5x)$, then $f'(x) = $ ___ _____

7. Find $\dfrac{dy}{dx}$ if $y = \dfrac{2x+7}{5-2x}$

     _____

8. Write the equation of the normal line to $y = 3^{x^2+1}$ when x = 1.

     _____

9. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = t^2 - 4t + 3$, where s is measured in meters and t is measured in seconds.

   a. Find the displacement of the particle during the first 2 seconds.

   b. Find the average velocity of the particle during the first 4 seconds

   c. Find the instantaneous velocity of the particle when t = 4.

   d. Find the acceleration of the particle when t = 4

   e. Describe the motion of the particle. When does it change directions? When is it speeding up?

Applications of Differentiation

1. At what value of x does the function $y = 1.2x^2 - e^{4x}$ change concavity? _____

2. Find all critical points for the function $f(x) = \frac{2}{3}x^3 + 5x^2 - 28x - 10$ _____

3. Let $y = \sin x$ on $\left[0, \frac{\pi}{2}\right]$. Approximate the value c guaranteed by the Mean Value Theorem. Please show all work to justify your conclusion. _____

4. A farmer has 160 meters of fence to enclose a rectangular area against a straight river. He only needs to fence in three sides. What is the maximum area that he can enclose with his materials?

_____

5. Write the equation of the tangent line and the normal line of $f(x) = x^3 + 2x - 1$ at the point (-1, 4).

_____

_____

6. Consider the function $f(x) = x^4 - x^3$

    a. Find the intervals on which f(x) is increasing or decreasing.

    b. Locate all extrema.

    c. Find the intervals on which f(x) is concave up or down.

    d. Find all points of inflection.

    e. Sketch the graph of f(x).

Integration

1. Evaluate the integral $\int \left(x^4 - 3x^2 + 1\right) dx$ _____

2. $\int \cos(3x)\, dx =$ _____

3. $\int \dfrac{5x^2 - 2x + 1}{x^2}\, dx =$ _____

4. $\int \dfrac{dx}{x \ln^5 x} =$ _____

5. $\int 2x\sqrt{1+x^2}\,dx$ _____

6. $\int_0^2 e^{x^2}\,dx$ _____

7. Find the total area of the region bounded by the curve $y = x^3$ and the x-axis on $[-1,1]$.

_____

8. The rate at which water flows out of a pipe is given by a differentiable function R(t). The table below records the rate at 4-hour intervals for a 24-hour period.

time (hours)	R(t) (gallons per hour)
0	9.4
4	10.1
8	10.6
12	10.9
16	10.7
20	10.2
24	9.5

a. Use the trapezoidal rule with 6 subintervals of equal length to approximate $\int_0^{24} R(t)\,dt$.

b. Explain the meaning of your answer in terms of the water flow, using correct units.

c.   Is there a time between 0 and 24 such that $R'(t) = 0$? Justify your answer.

d.   Suppose the rate of the water flow is approximated by $Q(t) = 0.01(940 + 24t - t^2)$. Use Q (t) to approximate the average water flow during the 24-hour period. Indicate units of measure.

9.   If $g(x) = \int_{\frac{\pi}{2}}^{x} \sin(3t)\,dt$, then $g'\left(\dfrac{\pi}{6}\right) =$ _____

10. The velocity of a particle moving along the x-axis is given by $v(t) = 2t^3 - 9t$ for $t \geq 0$.

a.   How far from the origin is the particle when t = 5?   _____

b.   How far has the particle traveled (in total) in the first 5 seconds?   _____

Polar Equations and Parametric Equations

1. Change from polar to rectangular form.

$$(-2, \pi)$$

           _____

2. Change from rectangular to polar form.

   a. $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2} \right)$

           _____

   b. $(8, -15)$

           _____

3. Change the rectangular equation to polar form.

   a. $x^2 + y^2 + 8x = 0$

           _____

b. $x^2 = 5y$ _____

4. Convert from polar form to an equation in rectangular form.

$r = 4\sin\theta$ _____

5. Obtain an equation in terms of x and y by eliminating the parameter. Then identify the curve.

a. $x = 2t$
$y = t^2$ _____

b. $x = 2 + 2\sin\theta$
$y = 3 + 2\cos\theta$ _____

Vectors and Dot Products

1. Given $a = \langle 5, -12 \rangle$ and $b = \langle -3, -6 \rangle$. Find the following:

   a. $a + b$ _____

   b. $2a + 2b$ _____

   c. $|a - b|$ _____

2. Find a unit vector that has the same direction as $-3i + 7j$. _____

3. What is the angle between the given vector and the positive direction of the x-axis?

   $i + \sqrt{3}\,j$ _____

4. Determine whether the given vectors are orthogonal, parallel or neither.

$$a = \langle 4,6 \rangle \qquad b = \langle -3,2 \rangle$$

_____

5. Find the angle between the vectors. Give your answer using an exact expression (no decimals).

$$a = \langle -2,5 \rangle \qquad\qquad b = \langle 5,12 \rangle$$

Sequences and Series

1. Write the first five terms of the sequence.

   a. $a_n = \sin\left(\dfrac{\pi n}{2}\right)$

   1a. _____

   b. b. $a_n = \left(-\dfrac{1}{4}\right)^n$

   1b. _____

2. Name the next two apparent terms of the sequence. Describe the pattern you used to find these terms.

   a. $5, 10, 20, 40, \ldots$

   2a. _____

   b. b. $3, -\dfrac{3}{2}, \dfrac{3}{4}, -\dfrac{3}{8}, \ldots$

   2b. _____

3. Simplify the ratio of factorials.

   3. _____

   $$\dfrac{(2n-1)!}{(2n+1)!}$$

4. Find the sum, if possible.

a. $\displaystyle\sum_{n=1}^{\infty} 2(0.9)^{n-1}$

4a. _____

b. $\displaystyle\sum_{n=1}^{50} 5n+3$

4b. _____

c. $\displaystyle\sum_{n=1}^{12} \left(n^3 - n + 2\right)$

4c. _____